Anisotropies in aggregates with biased random walks on two-dimensional lattices

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(Received 5 March 1993)

Irreversible stochastic models of growth in which particles are globally biased to seed are formulated and studied by Monte Carlo simulations on triangular, hexagonal, and square lattices. In the deterministic aggregation model (DAM), where only the effect of attraction between particles and the seed is considered, the particles have peculiar paths of motion on each lattice. Aggregates with biased random walks manifest special anisotropies on the two-dimensional lattices which are the same as those in DAM's. We discuss the universal and nonuniversal properties of these aggregates on the two-dimensional lattices.

PACS number(s): 64.60.Qb, 05.60.+w, 81.10.Dn

Even though the diffusion-limited aggregation [1] (DLA) has successfully described a broad range of nonequilibrium growths [2–6], it is not still well understood from a fundamental point of view. Furthermore, the motions of the particles in most studies on DLA and on similar random aggregates [2–6] have been restricted mainly to the ordinary random walks, i.e., the pure stochastic motions. In contrast, as in the case of electrodepositions, particles may be drived by some forces to the seed or to the cluster and the particles should do some deterministic motions as well as stochastic ones. To understand effects on such particle-cluster (particle-seed) attraction on random aggregates, several theoretical models [7–10] have recently been suggested. Meakin [7] has proposed a particle-drift DLA model and shown that the drift bias to a certain direction makes fractal dimensions of resulting clusters increase. Block, von Bloh, and Schnellnhuber [8] have suggested an off-lattice aggregation model with the particle-cluster attraction described by the power law (r^{-a}) and shown that fractal dimensions of clusters depend on the exponent a.

Recently we have studied an aggregation model [9] on a square lattice in which the motions of particles are biased random walks, otherwise the model is exactly the same as ordinary DLA [5]. In contrast to isotropic off-lattice aggregates [8], on a square lattice a combination [9] of both effects, one from the attraction and the other from the underlying lattice structure, makes aggregates anisotropic. The motions of the particles in our model [9] are biased to the seed as what follows. When a particle reaches a site on a lattice with the coordination number z, at the very next step the probability $P_{\max}$ to hop to one of four nearest neighbors (NN's), which is the shortest distance away from the seed, is set as

$$P_{\max}=\frac{1-\alpha}{z}+\alpha \quad (0\leq\alpha\leq 1)\ , \qquad (1)$$

and their probabilities ($P_{\min}$'s) to hop to the other $z-1$ NN's are set as

$$P_{\min}=\frac{1-\alpha}{z}\ . \qquad (2)$$

If $\alpha=0$, $P_{\max}=P_{\min}=1/z$, and our model reproduces the ordinary DLA [1,5]. If $\alpha=1$, the motion of a particle from the starting site to the deposition site on a square lattice ($z=4$) is deterministic and the main directions of the motion are along the directions $(\pm1,\pm1)$ on a square lattice because of the lattice structure [9]. We have called the model with $\alpha=1$ a deterministic aggregation model on a square lattice (SDAM). In SDAM the peculiar trajectories of the above-mentioned motion makes the aggregates $\times$-shaped and the effective fractal dimension D of these clusters is equal to 1 [9]. Physically interesting models should of course be the models with $0<\alpha<1$, which we have called aggregates with globally biased random walks on a square lattice (AGRS) [9]. We have investigated the crossover phenomena [9] from DLA ($\alpha=0$) to SDAM ($\alpha=1$) in AGRS by the variation of α and found that the clusters of AGRS are all $\times$-shaped and the effective dimension D of those clusters is 1. From these results it has been concluded that AGRS have the same critical property as that of SDAM and the crossover from DLA to SDAM in AGRS is sudden at $\alpha=0$[9]. The bias and the underlying lattice structure should make the clusters of AGRS anistropic and their fractal dimension 1. In the particle-drift DLA model by Meakin [7] on lattices the drift to a given direction makes the fractal dimension of clusters greater than that of ordinary DLA. In contrast the bias [9] to the seed on a square lattice makes clusters anisotropic and slim.

Ordinary large-scale square-lattice DLA's are anisotropic and starlike [5], but on triangular and hexagonal lattices the resulting clusters are rather isotropic [11]. In a sense DLA on a square-lattice should be in a different universality class from DLA's on triangular and hexagonal lattices. We think that it is therefore very interesting to check whether the anisotropy in AGRS is common for aggregates with biased RW's on the various two-dimensional lattices or not. It is the motivation of this study to investigate what properties of AGR are universal on the various two-dimensional lattices and what properties depend on the specific lattice structure. Let us now think about the generalizations of our model [9] on an arbitrary two-dimensional lattice. To be specific, let us first think about the deterministic aggregation model ($\alpha=1$) on a triangular lattice (TDAM) with $z=6$. A typical trajectory of motion of a particle in TDAM is shown by the bold line in Fig. 1. When a particle starts from a site on the starting circle, under the rules from Eq. (1)

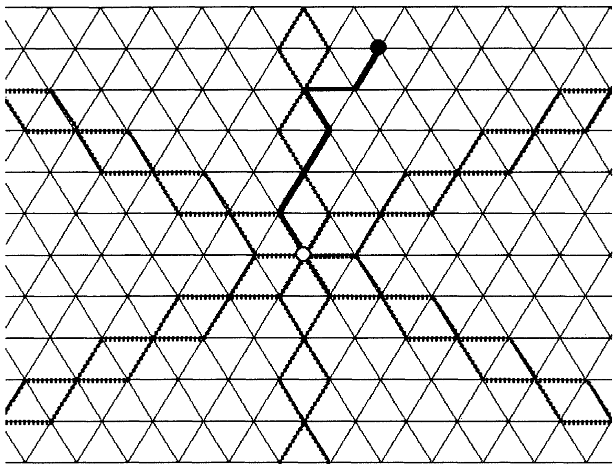

FIG. 1. Bold lines which start from a starting site (filled circle) to the seed (unfilled circle) show a typical trajectory of a particle in the deterministic aggregation model on a triangular lattice (TDAM). Siz major axes in TDAM are noted by bold-dotted lines.

and (2) with $\alpha=1$, it first moves straight to a site on the nearest major axis from the starting position. Then it moves on a zigzag path along the axis to the seed if there are not any already aggregated particles in that path. Such six major axes on a triangular lattice are denoted by bold-dotted lines in Fig. 1. As shown in Fig. 2(a), the major axes in TDAM are along lines which connect the seed site to six nearest sites on the hexagonal lattice which is dual to the original triangular lattice. In TDAM at the initial stage when only a few particles are aggregated the cluster is $*$-shaped similar to one in Fig. 3. The starting positions of the particles are then categorized into ($A-F$) regions and 1–6 regions as in Fig. 3. The particles start-

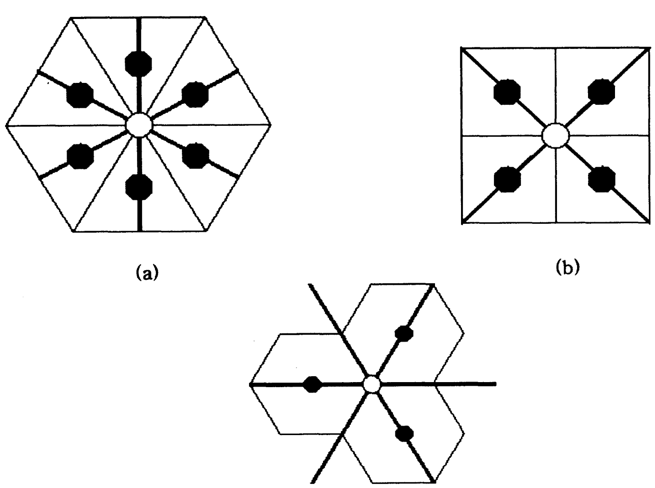

FIG. 2. Major axes of deterministic aggregation models (DAM's) on various lattices. Unfilled circles are seed sites and filled circles are nearest sites on dual lattices to the original lattices. Directions of main axes are along the bold lines. (a) DAM on a triangular lattice (TDAM). (b) DAM on a square lattice (SDAM). (c) DAM on a hexagonal lattice (HDAM).

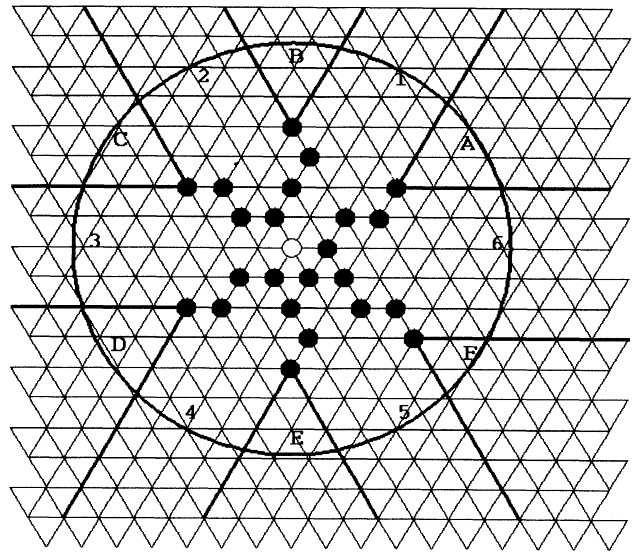

FIG. 3. A categorization of starting positions of particles on a circle centered at the seed (unfilled circle) in TDAM. Filled circles denote the already aggregated particles in TDAM. Particles from the A–F regions eventually enlongate the six main branches but particles from the 1–6 regions only thicken six main branches.

ing from the ($A-F$) regions eventually come to the cluster with zigzag paths along one of the major axes and they are aggregated to the cluster in directions to enlongate the main branches of the $*$-shaped clusters. But the particles from the 1–6 regions are aggregated in directions to thicken the main branches. In a theory where the radius of the starting circle is ∞, the particles coming from the 1–6 regions are negligible compared to those from the $A-F$ regions. The clusters of TDAM are therefore very thin $*$-shaped clusters and the fractal dimension D of these is 1. But in the computer simulations or in experiments the radius cannot be made ∞, and there should be definite contributions of particles from the 1–6 regions. In simulations this effect should be handled very carefully, otherwise we might see some artifacts. On the two-dimensional lattices deterministic aggregation models, (DAM's) ($\alpha=1$) have a universal property that the effective fractal dimension of the clusters is 1, but the shapes of clusters and the number of main branches of clusters depend on the lattice structure. The major axes of a DAM on a square lattice (SDAM) are along lines which join the seed to four nearest sites on the square lattice which is dual to the original square lattice [9]. [See Fig. 2(b).] The major axes of the DAM on a hexagonal lattice (HDAM) are along lines which join the seed to three nearest sites on the triangular lattice which is dual to the original hexagonal lattice. [See Fig. 2(c).] The clusters of HDAM are also rotated $*$-shaped clusters and the effective fractal dimension D is 1.

Now let us discuss simulations and their results for aggregates with globally biased random walks (RW's) ($0<\alpha<1$) (AGR) in which the motions of particles are biased RW's. Particles start from random points on a circle centered at the seed with a radius $r_{max}+R_s$, where r_{max} is the distance between the seed and the most distant aggregated particle from the seed (i.e., the maximum radius of the cluster). In the ordinary DLA [5], R_s is safe enough to be as small as 5. As warned when introducing TDAM by use of Fig. 3, simulations for AGR on a triangular lattice (AGRT) may show some artifacts if the radius of the starting circle is too small. To see the effect of bias correctly, R_s should be sufficiently large. In order to avoid this starting-radius problem and to see the trend which depends on the variation of R_s, we have done simulations for several different R_s's for AGRT with given α. The results for simulations for AGRT with $\alpha=0.05$ are shown in Figs. 4 and 5. A typical cluster with $r_{\max}=400$ of AGRT with $\alpha=0.05$ and $R_s=400$ is shown in Fig. 4(a). As you can see from Fig. 4(a) the cluster is grown mainly along the same six major axes explained in Fig. 2(a) and the shape of the cluster manifests anisotropy clearly. A cluster from different $R_s(=100)$ is shown in Fig. 4(b). Comparing the cluster in Fig. 4(a) to one in Fig. 4(b), we can infer that for the same value of α the width of main branches gets slimmer when R_s gets larger. A method to study a quantitative property of clusters is to measure the number of particles $N(r)$ within a distance r from the seed. If clusters are statistically self-similar fractals, then $N(r)$ is expected to satisfy the relation

$$N(r)=C(\alpha)r^D \tag{3}$$

or

$$\ln N(r)=D\ln r+\ln C(\alpha) \tag{4}$$

for a suitable range of r, where D is the fractal dimension of cluster and $C(\alpha)$ is a constant independent of r. Figure 5 shows the dependence of $\ln N(r)$ on $\ln r$ for the clusters of AGRT with $\alpha=0.05$ and $R_s=400$, and that with $R_s=100$. At least ten clusters have been used for each curve in Fig. 5. We can expect that for the smaller value for R_s, the thicker $*$-shaped cluster occurs. As we can see in Figs. 4(a) and 4(b), the clusters are anisotropic and thus the plot of $\ln N(r)$ vs $\ln r$ in Fig. 5 cannot be a strictly straight line. However, by fitting the relation (4) to the data in Fig. 5, we can identify the effective fractal dimension D, from which we infer a quantitative property of clusters. From data in Fig. 5 in the range $50\leq r\leq 200$, it is found that $D=1.50\pm0.02$ for AGRT with $\alpha=0.05$ and $R_s=100$, and $D=1.11\pm0.05$ for AGRT with $\alpha=0.05$ and $R_s=400$, respectively. We have also done simulations for AGRT with $\alpha=0.05$ and $R_s=30$ and got $*$-shaped clusters with fat branches and $D=1.73\pm0.01$. From the results for different R_s's we can conclude that if the radius of the starting circle gets larger, the branches get slimmer and the effective dimension D becomes smaller. We thus believe that AGRT for arbitrary small α (if

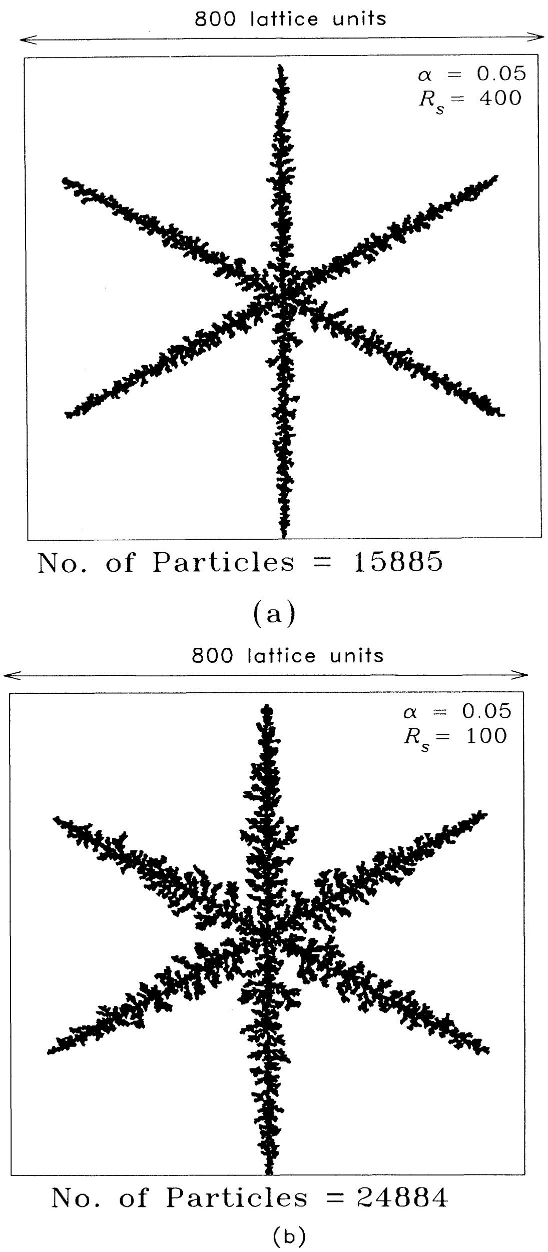

FIG. 4. (a) A typical cluster of AGRT with $\alpha=0.05$ and $R_s=400$. (b) A typical cluster of AGRT with $\alpha=0.05$ and $R_s=100$.

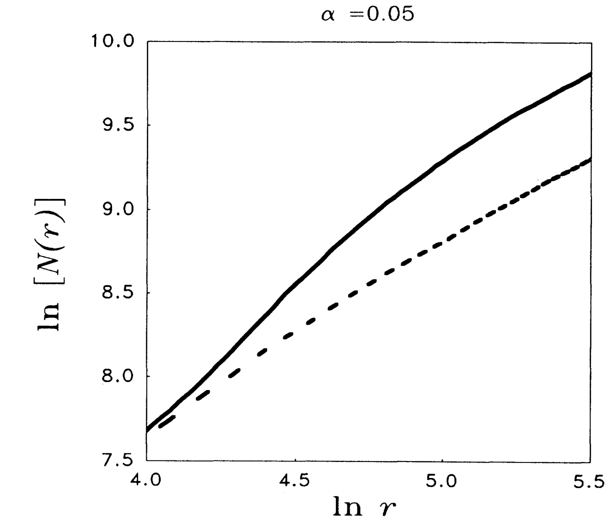

FIG. 5. Dependence of $\ln N(r)$ on $\ln r$ for the clusters for AGRT with $\alpha=0.05$. The normal curve is for clusters with $R_s=100$. The dashed curve is for clusters with $R_s=400$.

$\alpha \neq 0$) is quite different from ordinary DLA on a triangular lattice [11] and the clusters of AGRT are $*$-shaped cluster, which is nearly the same as that of TDAM. We have also done simulations for AGRT with $\alpha = 0.5$ and $R_s = 100$ and gotten very slim $*$-shaped clusters with $D = 1.0 \pm 0.02$. We can conclude from these results that the property of AGRT is the same as that of TDAM if $R_s \rightarrow \infty$ and the crossover from DLA to AGRT is very sudden at $\alpha = 0$. Even for finite R_s, we can see the branches of $*$-shaped clusters clearly, but the widths of the branches get slimmer (or $D \rightarrow 1$) as R_s gets larger and α gets larger.

We have also done similar simulations for aggregates with globally biased RW's on a hexagonal lattice (AGRH). A typical cluster of $r_{max} = 400$ for AGRH with $\alpha = 0.05$ and $R_s = 200$ is shown in Fig. 6. As you can see from Fig. 6 the cluster is grown along the same directions as explained in Fig. 2(c). We have also done simulations for AGRH with $\alpha = 0.05$ and $R_s = 400$ and gotten similar results to those of the corresponding AGRT, except the directions of main branches. On a hexagonal lattice we can also conclude that the property of AGRH is the same as that of HDAM if $R_s \rightarrow \infty$ and the crossover from DLA to AGRH is very sudden at $\alpha = 0$.

The conclusions of this paper are as follows. (i) On two-dimensional lattices, the critical properties of AGR are universally the same as those of the corresponding DAM's. (ii) Clusters of AGR are anisotropic and the directions of main branches depend on lattice structures. The universality in the directions of branches is that they are along the lines which join the seed to the nearest sites on the dual lattice to the original lattice. (iii) The crossover of AGR from DLA to DAM is very sudden at $\alpha = 0$ and the main branches do exist if $\alpha \neq 0$. (iv) One can make as many main branches in the clusters of AGR as one wants if one chooses a proper lattice. If you choose suitable R_s's, you can control the width of main branches of AGR on a given lattice.

The final discussion is on a theoretical conjecture on AGR. According to the tip theory of growth of fractal patterns [12–14] the relation $dR/dN = cp(R)$ holds, where R is the length of a tip of a fractal pattern, N is the number of particles in a fractal pattern, $p(R)$ is the probability that a particle hits the tip, and c is a constant which depends on the particle size or other geometrical factors. In AGR the tip should be the end of main branches. The bias $\alpha(\neq 0)$ and the lattice structure should make particle trajectories hit at the ends of main branches with some finite probability as in the case of DAM and thus $p(R) = p(\alpha)$. This means that $p(R)$ is independent of the length R and proves that $R = cp(\alpha)N$ and $D = 1$. If this picture is right, the bias to the seed is the main theoretical reason for the slim branches.

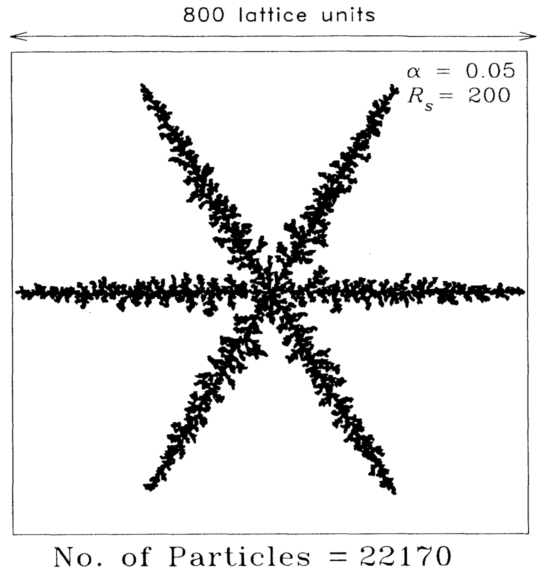

FIG. 6. A typical cluster of AGRH with $\alpha = 0.05$ and $R_s = 200$.

The authors thank S. H. Park and D. K. Park for helping with computer programming. This work was supported in part by Korea Sanhak Foundation, by Basic Science Research Institute Program, Ministry of Education, Republic of Korea, and by KOSEF through the Center for Thermal and Statistical Physics in Korea University.

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